WNE Linear Algebra Final Exam
Series A

9 February 2017

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem.**

Problem 1.
Let $v_1 = (1,1,1,1), v_2 = (2,1,2,3), v_3 = (1,0,1,t)$ be vectors in $\mathbb{R}^4$.

a) for which $t \in \mathbb{R}$ vectors $v_1, v_2, v_3 \in \mathbb{R}^4$ are linearly independent?
b) find a system of linear equations which set of solutions is equal to $\text{lin}(v_1, v_2, v_3)$ for $t = 3$.

Problem 2.
Let $W \subset \mathbb{R}^5$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + x_2 + 2x_3 - x_4 + 2x_5 = 0 \\ x_1 + x_2 + 3x_3 + x_4 + 3x_5 = 0 \\ 2x_1 + 3x_2 + 5x_3 - 3x_4 + 3x_5 = 0 \end{cases}$$

a) find a basis $\mathcal{A}$ of the subspace W and the dimension of W,
b) complete the basis $\mathcal{A}$ to a basis $\mathcal{B}$ of $\mathbb{R}^5$ and find coordinates of $w = (1,0,0,0,0) \in \mathbb{R}^5$ relative to $\mathcal{B}$.

Problem 3.
Let $\varphi \colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by the formula

$$\varphi((x_1, x_2, x_3)) = (-4x_1 + x_2 + 2x_3, tx_2, -x_1 + x_2 - x_3).$$

a) for $t = -3$ find matrix $C \in M(3 \times 3; \mathbb{R})$ such that matrix $C^{-1}M(\varphi)_{st}^{st}C$ is diagonal,
b) find all $t \in \mathbb{R}$ for which there exist a basis $\mathcal{A}$ of $\mathbb{R}^3$ such that $M(\varphi)_{\mathcal{A}}^{\mathcal{A}} = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & q \end{bmatrix}$, where $p, q \in \mathbb{R}$.

Problem 4.
Let $\mathcal{A} = ((1,1,0),(0,0,1),(2,3,0))$ be an ordered basis of $\mathbb{R}^3$. The linear transformation $\psi \colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is given by the matrix $M(\psi)_{\mathcal{A}}^{\mathcal{A}} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$.

a) find $M(\psi)_{\mathcal{A}}^{st}$,
b) find formula of $\psi \circ \psi$.

Problem 5.
Let $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 - x_2 + 2x_3 = 0\}$ be a subspace of $\mathbb{R}^3$.

a) find an orthonormal basis of $V^\perp$,
b) compute the orthogonal projection of $w = (3, 0, 0)$ onto V.

Problem 6.
Let

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

a) compute matrix AB,
b) compute $\det(B^4 A^{-1} + B^5)$.

Problem 7.
Let $L \subset \mathbb{R}^3$ be an affine line given by the system of linear equations

$$\begin{cases} x_1 - x_3 = 2 \\ 2x_1 - x_2 = 3 \end{cases}$$

a) find a parametrization of L,
b) find an equation of the affine plane perpendicular to L passing through $(1, 0, 0)$.

Problem 8.
Consider the following linear programming problem $-4x_1 - 3x_2 + 5x_3 - 2x_5 \to \min$ in the standard form with constraints

$$\begin{cases} x_1 + x_2 - x_3 + x_4 = 3 \\ 2x_1 + x_2 - 2x_3 + x_5 = 4 \end{cases} \quad \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 5$$

a) which of the sets $\mathcal{B}_1 = \{1, 3\}, \mathcal{B}_2 = \{2, 3\}, \mathcal{B}_3 = \{4, 5\}$ are basic? Which basic sets are feasible?
b) solve the linear programming problem using simplex method.